

# Description Logic Rules

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## Class expressions

class names	$A, B$
conjunction	$C \sqcap D$
disjunction	$C \sqcup D$
negation	$\neg C$
exist. role restr.	$\exists R.C$
univ. role restr.	$\forall R.C$
Self	$\exists S.\text{Self}$
atleast	$\geq n S.C$
atmost	$\leq n S.C$
nominal	$\{a\}$

## Roles

role names	$R, S, T$
simple roles	$S, T$
inverse roles	$R^{-}$
universal role	$U$

## Tbox (Class axioms)

inclusion	$C \sqsubseteq D$
equivalence	$C \equiv D$

## Rbox (Role axioms)

inclusion	$R_1 \sqsubseteq R_2$
RIA	$R_1^{(-)} \circ \dots \circ R_n^{(-)} \sqsubseteq R$
transitivity	$\text{Tra}(R)$
symmetry	$\text{Sym}(R)$
reflexivity	$\text{Ref}(R)$
irreflexivity	$\text{Irr}(S)$
disjointness	$\text{Dis}(S, T)$

## Abox (Facts)

class membership	$C(a)$
role relation	$R(a, b)$
neg. role relation	$\neg S(a, b)$
equality	$a \approx b$
inequality	$a \not\approx b$

- **First order logic rules:**

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$Woman(x) \vee Man(x) \leftarrow Person(x)$	(FOL clause)
$Father(x) \leftarrow Man(x) \wedge hasChild(x, y)$	(definite Horn clause, Datalog rule)
$hasUncle(x, y) \leftarrow hasBrother(mother(x), y)$	(+function symbol)
$\leftarrow Man(x) \wedge Woman(x)$	(Horn clause, “integrity constraint”)
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- **Tight integration with DL: SWRL rules**
- **Many other DL+rules approaches:**  
HEX programs, MKNF, WFS, ... (off topic here)

**Problem:** DL + SWRL is undecidable (explain why)

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## Description Logic Programs – DLP

- “Rules fragment of OWL DL”, e.g.  
 $C \sqsubseteq \forall R.D \rightsquigarrow C(x) \wedge R(x, y) \rightarrow D(y)$
- easy to support in reasoning
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## DL-safe rules

- restrict Datalog rules to *named individuals*
- limited interaction with DL
- supported only by Pellet and KAON2

*SROIQ* can do more than DLP!

## Example

$$\text{Man}(x) \wedge \text{hasChild}(x, y) \rightarrow \text{fatherOf}(x, y)$$

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$\rightsquigarrow$  DL knowledge base:

$$\begin{array}{l} \text{Man} \sqsubseteq \exists \text{man}.\text{Self} \quad (\text{Tbox}) \\ \text{man} \circ \text{hasChild} \sqsubseteq \text{fatherOf} \quad (\text{Rbox}) \end{array}$$

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$\rightsquigarrow$  **DL rules**: rules that can be expressed in *SROIQ*

## Further examples

- $\text{Elephant}(x) \wedge \text{Mouse}(y) \rightarrow \text{biggerThan}(x, y)$  (concept product)
- $\text{Man}(x) \wedge \text{hasBrother}(x, y) \wedge \text{hasChild}(y, z) \rightarrow \text{Uncle}(x)$
- $\text{marriedTo}(x, y) \wedge \text{loves}(x, y) \rightarrow \text{Happy}(x)$   
(simple role conjunctions – need some work)

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Mix ingredients and blend well, add  $n$  variable DL-safe Datalog to taste

# An ELP rule base

- (1)  $\text{NutAllergic}(x) \wedge \text{NutProduct}(y) \rightarrow \text{dislikes}(x, y)$
- (2)  $\text{Vegetarian}(x) \wedge \text{FishProduct}(y) \rightarrow \text{dislikes}(x, y)$
- (3)  $\text{orderedDish}(x, y) \wedge \text{dislikes}(x, y) \rightarrow \text{Unhappy}(x)$
- (4)  $\text{dislikes}(x, v) \wedge \text{Dish}(y) \wedge \text{contains}(y, v) \rightarrow \text{dislikes}(x, y)$
- (5)  $\text{orderedDish}(x, y) \rightarrow \text{Dish}(y)$
- (6)  $\text{ThaiCurry}(x) \rightarrow \text{contains}(x, \text{peanutOil})$
- (7)  $\text{ThaiCurry}(x) \rightarrow \exists \text{contains.FishProduct}(x)$
- (8)  $\rightarrow \text{NutProduct}(\text{peanutOil})$
- (9)  $\rightarrow \text{NutAllergic}(\text{sebastian})$
- (10)  $\rightarrow \exists \text{orderedDish.ThaiCurry}(\text{sebastian})$
- (11)  $\rightarrow \text{Vegetarian}(\text{markus})$
- (12)  $\rightarrow \exists \text{orderedDish.ThaiCurry}(\text{markus})$

( $v$  is a safe variable)

## DL rules

- non-boring FOL rule fragment of *SROIQ*
- full reasoning support in any OWL 2 reasoner
- no impact on worst-case complexity even in smaller DLs

## ELP

- expressive, tractable, rule-based, DL-compatible (& non-trivial)
- support for polytime compilation into Datalog
- DLP,  $\mathcal{EL}^{++}$ , and DL-safe rules in one language

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